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# Equilibrium statistical mechanics and energy partition for the shallow water model

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## Abstract

Geophysical flows are highly turbulent, and yet present large-scale coherent structures. **Statistical mechanics** is a powerful approach allowing to describe this **self-organization** phenomenon with only a few thermodynamical parameters such as the energy and the enstrophy of the flow. It has been applied with success to several geophysical problems, such as the description of jovian and oceanic vortices. However, due to important technical difficulties, those previous applications were, up to now, limited to the use of quasi-geostrophic models, which do not contain **inertia-gravity waves**. Here, [1], we generalize the statistical mechanics theory to the **shallow water model**, which is the simplest model allowing for the presence of inertia-gravity waves. This model captures a **direct energy cascade** through wave motions and shocks, **along with** a concomitant **inverse energy cascade** of the vortical part of the flow. Within the statistical mechanics framework, **we predict** the emergence of a **balanced large scale flow** that contains only **a fraction of the total energy**. The excess energy is stored among **small scale fluctuations that can be interpreted as inertia-gravity waves**. The theory thus predicts the fraction of energy dissipated through interactions between the vortical part and the wave part of the dynamics.

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