Geometric focusing of internal waves: linear theory

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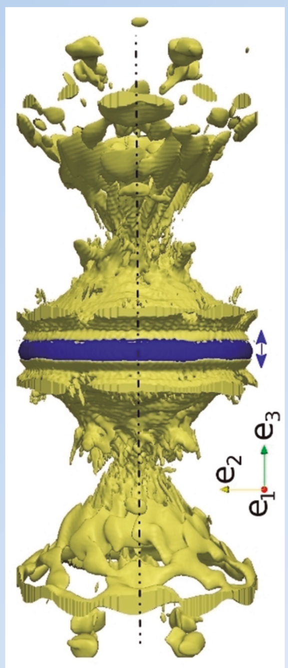
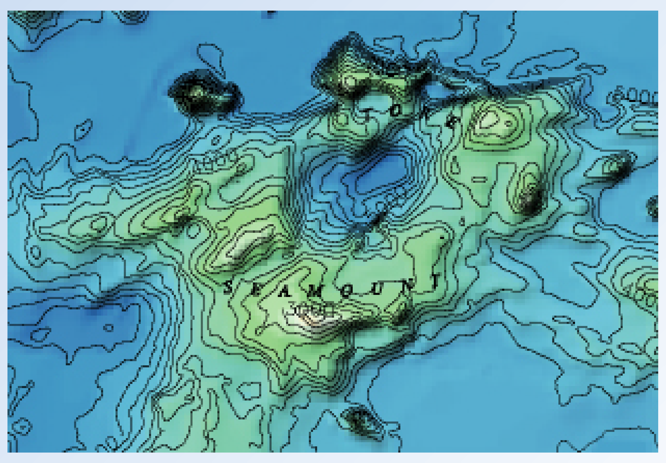
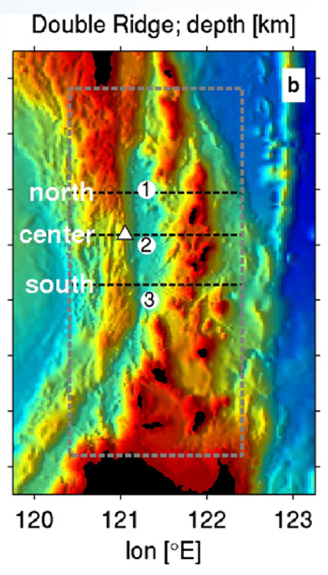
Motivation





Tore Seamount

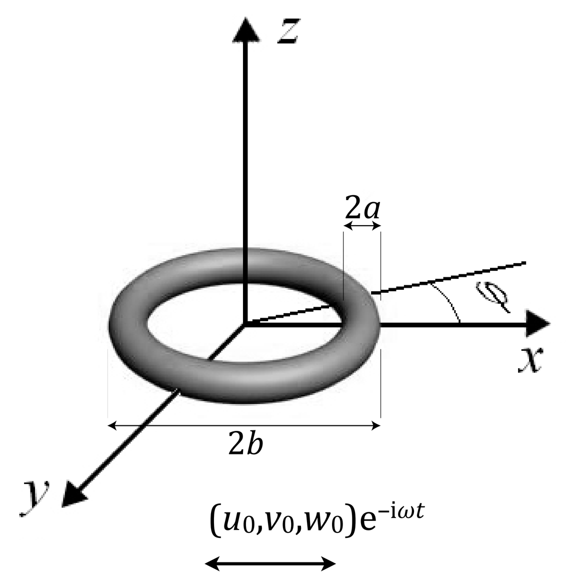


Luzon Strait

- A new, three-dimensional type of focusing
- Taking place away from boundaries
- Relevant to curved oceanic ridges

(Duran-Matute et al. *PRE* 2014)

Theoretical outline



A horizontal circular annulus oscillates in a stratified fluid of frequency N and viscosity ν

- Generation parameters

$$\epsilon = \frac{b}{a},\quad \Omega = \frac{\omega}{N},\quad St = \frac{\omega a^2}{\nu},\quad Ke = \frac{|\boldsymbol{u}_0|}{\omega a}$$

- Propagation parameters

$$\theta = \arccos\left(\frac{\omega}{N}\right),\quad \beta = \frac{\nu}{2\omega\tan\theta}$$

Taken separately, each cross-section admits a two-dimensional representation

$$q(x,z) \equiv \nabla\cdot\boldsymbol{u} = u_0\frac{\partial}{\partial x}f(x,z) + w_0\frac{\partial}{\partial z}g(x,z)$$

This representation remains valid for a slender annulus (ε » 1)

$$q(x,y,z) = \left(u_0\frac{\partial}{\partial x} + v_0\frac{\partial}{\partial y}\right)f(r_h - b, z;\varphi) + w_0\frac{\partial}{\partial z}g(r_h - b, z;\varphi)$$

A uniform approximation of its spectrum is built, combining at scale b the global interference of all the sections, and at scale a the local structure of each section, in terms of the radial and azimuthal Fourier components

$$f_n^{(c,s)}(k,z) = \frac{1}{2\pi}\int_{-\pi}^{\pi}d\varphi\int_{-\infty}^{\infty}dx\, f(x,z;\varphi)(\cos,\sin)(kx)e^{-in\varphi}$$

The vertical velocity follows as (and similarly for the other components)

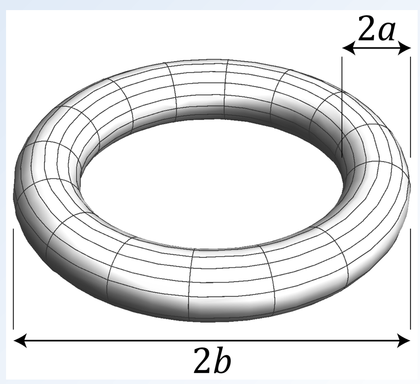
$$w = \frac{b}{4}\cos^2\theta e^{-i\omega t}\,\mathrm{sign}\,z\sum_{n=-\infty}^{\infty}e^{in\varphi}\int_0^{\infty}d\kappa\,\kappa^2 A_n(\kappa)\exp\left(-\frac{\beta\kappa^3|z|}{\cos\theta}\right)J_n(\kappa r_h\cos\theta)\exp(-i\kappa|z|\sin\theta)$$

with

$$\begin{aligned}A_n(\kappa) = &-2iw_0\sin\theta\,\mathrm{sign}\,z\left[J_n(\kappa b\cos\theta)g_n^{(c)} - Y_n(\kappa b\cos\theta)g_n^{(s)}\right](\kappa\cos\theta, -\kappa\sin\theta\,\mathrm{sign}\,z)\\ &+ (u_0 + iv_0)\cos\theta\left[J_{n+1}(\kappa b\cos\theta)f_{n+1}^{(c)} - Y_{n+1}(\kappa b\cos\theta)f_{n+1}^{(s)}\right](\kappa\cos\theta, -\kappa\sin\theta\,\mathrm{sign}\,z)\\ &- (u_0 - iv_0)\cos\theta\left[J_{n-1}(\kappa b\cos\theta)f_{n-1}^{(c)} - Y_{n-1}(\kappa b\cos\theta)f_{n-1}^{(s)}\right](\kappa\cos\theta, -\kappa\sin\theta\,\mathrm{sign}\,z)\end{aligned}$$

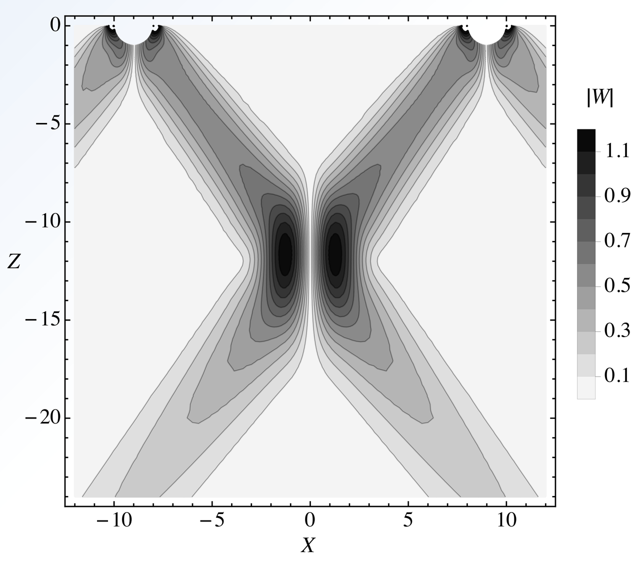
Full torus

$$q(x,z) = (1 + i\tan\theta)u_0\frac{x}{a}\delta\left(\sqrt{x^2+z^2} - a\right)$$

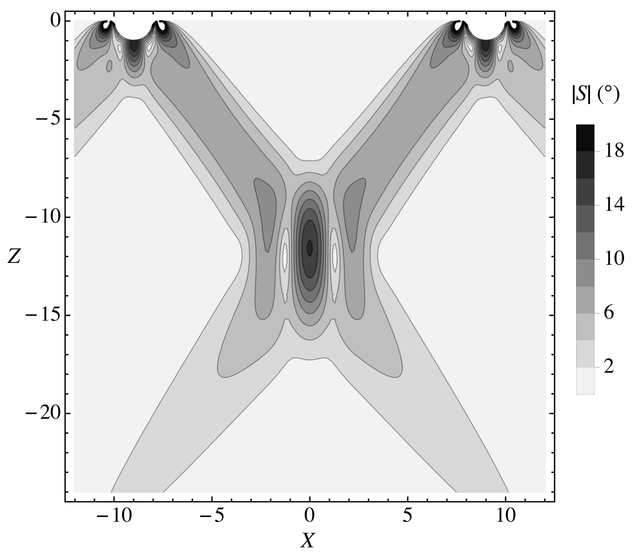


$\varepsilon = 9,\ \Omega = 0.8$

$St = 120,\ Ke = 0.2$



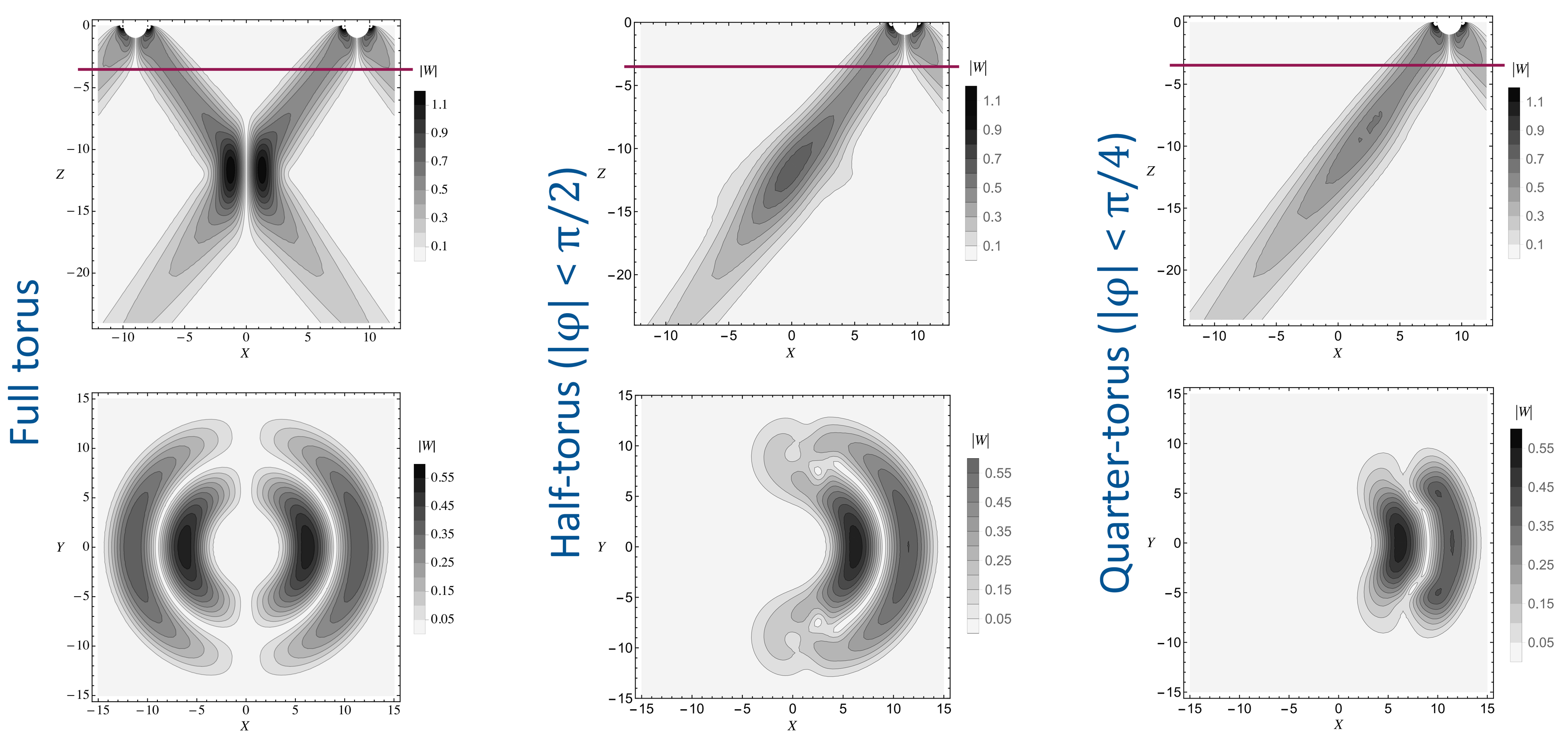
Vertical velocity



Isopycnic slope

Focusing and significant slopes even at low oscillation amplitude *Ke*

Partial tori



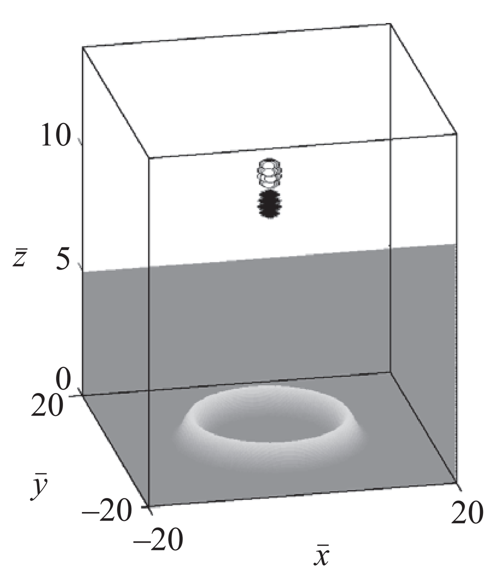
Along the vertical

Along the horizontal

Focusing persists, with weaker intensity, stressing the importance of horizontal curvature

Gaussian ring

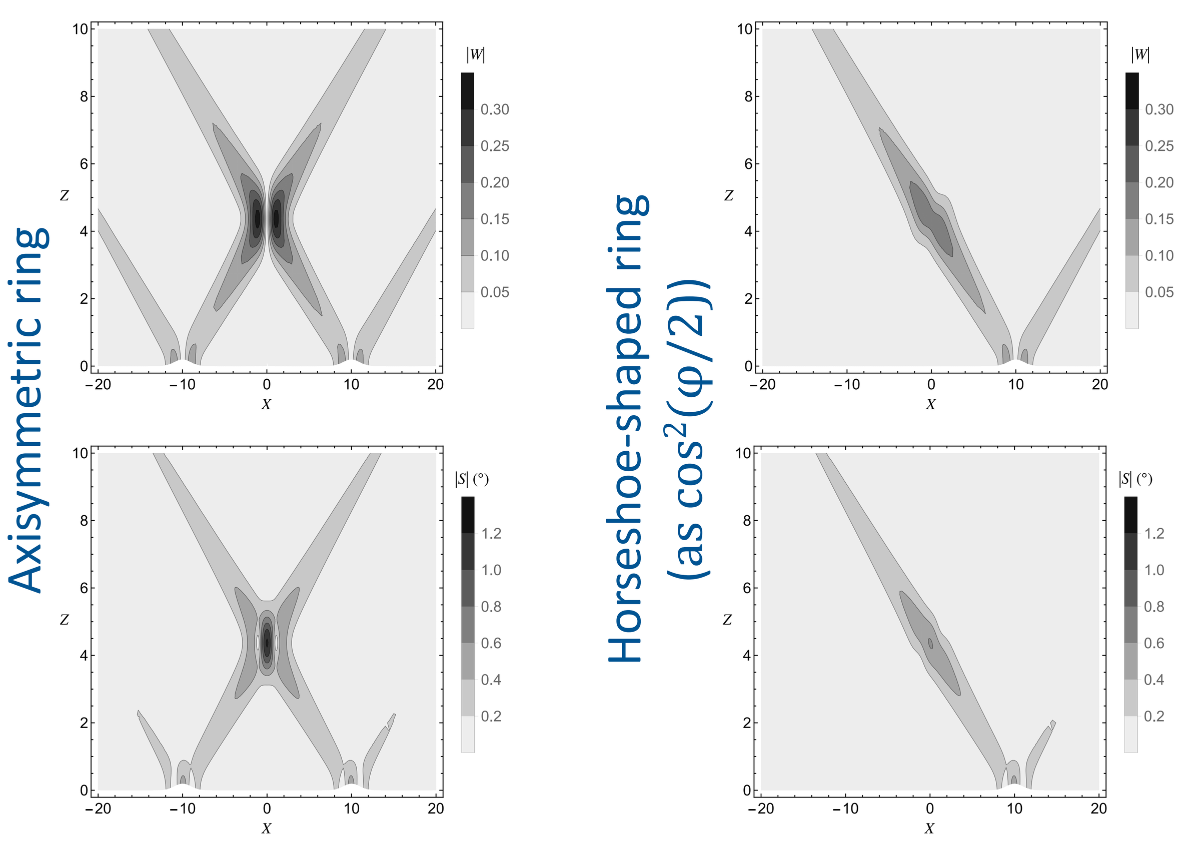
$$q(x,z) = 2u_0 h\delta(z)\frac{\partial}{\partial x}\exp\left(-\frac{x^2}{2a^2}\right)$$



(Bühler & Muller *JFM* 2007; Grisouard & Bühler *JFM* 2012)

$\varepsilon = 10,\ h/a = 0.2,\ \Omega = 0.4$

$St = \infty,\ Ke = 0.04$



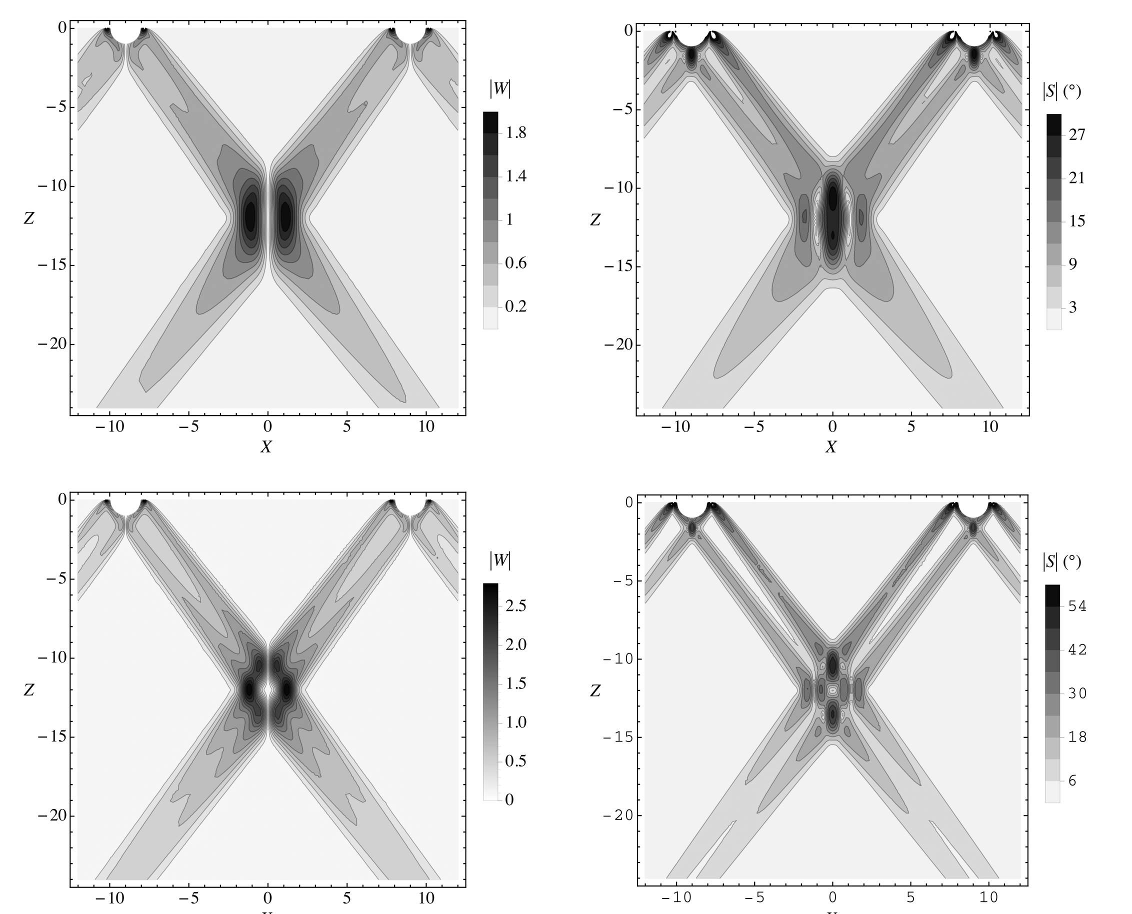
Velocity

Slope

Focusing does not rely on criticality

Modality and focusing

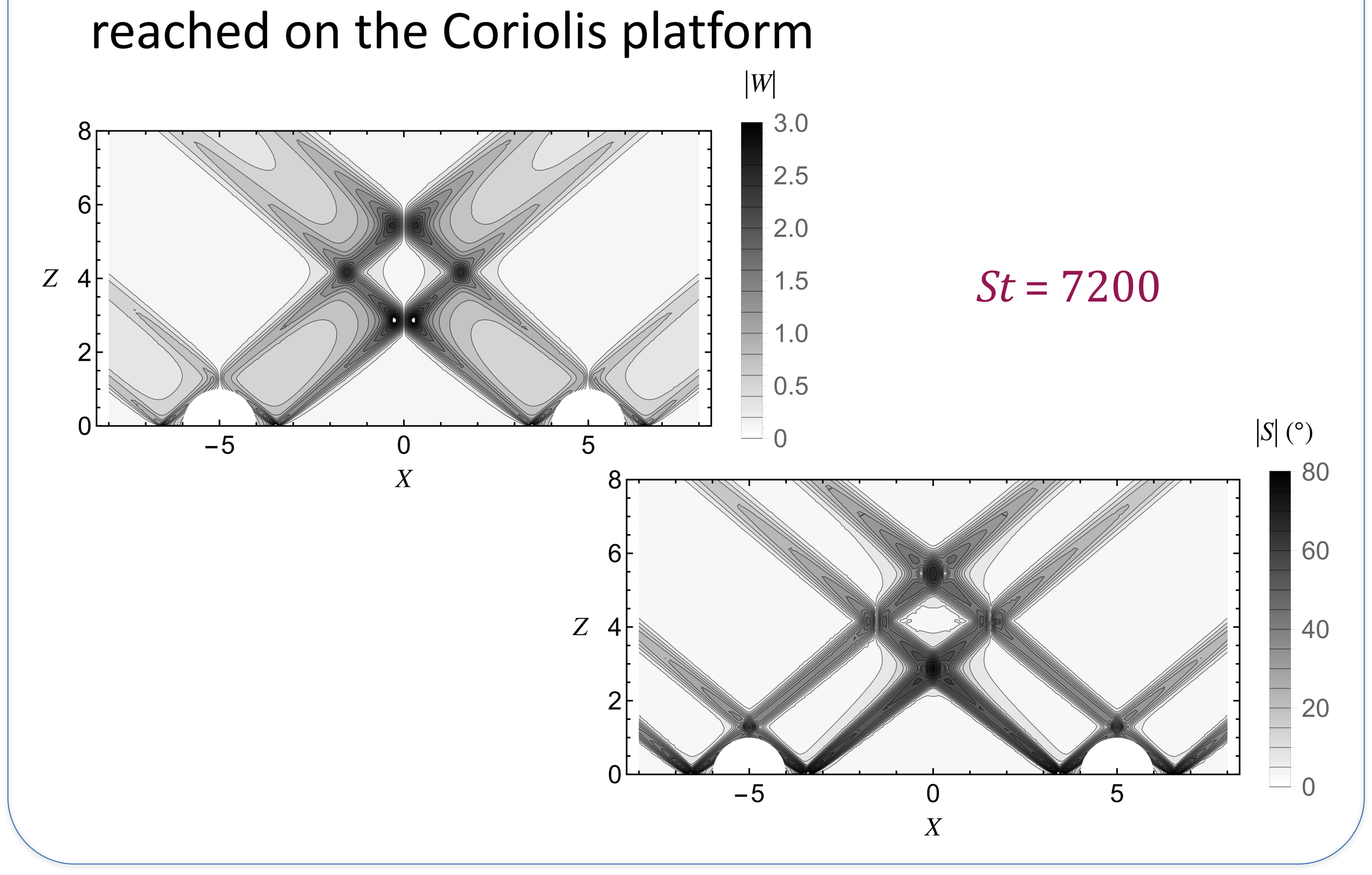
- As *St* increases and the waves evolve from unimodal to bimodal, smaller structures and higher slopes develop



St = 470

St = 1900

- Up to the scales and fully bimodal regime that can be reached on the Coriolis platform



St = 7200